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ABSTRACT

This paper clarifies the relationship between the absorbed crush energy and the dissipated crush energy and explores the use of each in crush and conservation of energy analysis. There is inconsistency and confusion in the literature of accident reconstruction regarding when crush analysis and conservation of energy analysis should use the *absorbed* crush energy and when it should use the *dissipated* crush energy. It is demonstrated in this paper that crush analysis calls for the absorbed energy, while conservation of energy analysis calls for the dissipated energy. However, this paper also shows that the equations of crush analysis and conservation of energy analysis can be written in terms of *either* the absorbed or the dissipated crush energies, since the absorbed and dissipated energies are related through the coefficient of restitution (when friction-type energy losses are assumed negligible). The assumptions of crush analysis are explored in order to develop a consistent approach.

CLARIFYING THE USE OF THE ABSORBED AND DISSIPATED ENERGIES

Equations (1) and (2) are the well-known crush analysis equations that relate the crush energy to the approach velocity and the vehicle changes in velocity for a central impact [8].

$$V_A = \sqrt{\frac{M_1 + M_2}{M_1 M_2} \cdot 2 \cdot E} \qquad (1)$$

$$\Delta V_i = \frac{1}{M_i}\sqrt{\frac{M_1 M_2}{M_1 + M_2} \cdot 2 \cdot E} \qquad (2)$$

In Equations (1) and (2), V_A is the relative approach velocity at impact, ΔV_i is the approach phase velocity change for the vehicle under consideration (i = 1,2), M_1 and M_2 are the vehicle masses, and E is the total crush energy for the impact. Equation (2) does not account for the ΔV that occurs during the restitution phase of the impact.

There is inconsistency and confusion in the literature of accident reconstruction regarding whether Equations (1) and (2) should employ the *absorbed* crush energy E_A or the *dissipated* crush energy E_d.[1,2,3,4] The absorbed energy is defined as the system deformation energy at the point of maximum dynamic crush. Prior to the end of the impact, the structure rebounds partially and restores some of the absorbed deformation energy back to the vehicles in the form of kinetic energy. The system deformation energy at the point the vehicles separate, after the partial restoration of energy, is the dissipated energy.[5]

To clarify the relationship of the absorbed crush energy to the dissipated crush energy and to explore the use of each in Equations (1) and (2), consider the physics of a barrier impact crash test. During a barrier impact,

[1] In Reference 3, Carpenter and Welcher observed, "Review of the literature related to collision energy analysis reveals numerous works with conflicting and, in some cases, incorrect usage of the terms energy absorption, energy recovery and energy dissipation." In their paper, they clarify the difference between the absorbed and dissipated energies in a manner consistent with the discussion contained in this paper. However, Carpenter and Welcher continue, "The residual deformation to a vehicle only correlates with the dissipated energy, not the absorbed energy. Traditional accident reconstruction methods utilizing energy and post-crash crush data are analyzing the relationship between the dissipated energy and the BEV or Delta-V, not the absorbed energy." As will be discussed below, it is actually the absorbed energy that correlates to the ΔV and the BEV.

[2] In Reference 7, Happer, et al, cite an equation identical to Equation (25) below, with the exception that they use the absorbed energy instead of the dissipated energy. It will be shown below that the role of the coefficient of restitution in Equation (25) is to convert the dissipated energy into the absorbed energy, and so, it is actually the dissipated energy that should be used in this equation.

[3] Reference 6 also confuses the use of the terms *absorbed energy* and *dissipated energy*. For instance, on Page 70-16, the narrative reads as follows: "The G term can be thought of as the energy dissipated without permanent damage." As the discussion that follows will show, this statement should have been given in terms of the absorbed energy, not the dissipated energy.

[4] There is nothing unique about the examples given in the first three footnotes. The intent of these footnotes is simply to offer representative examples, not to single out these specific publications as unusual.

[5] This statement is true when friction-type energy losses are assumed negligible. The derivation of the crush analysis equations [Equations (1) and (2)] makes this assumption and it is employed throughout this paper.

crushing of the vehicle structure absorbs the vehicle's initial kinetic energy. This absorption of energy occurs during the approach phase of the impact as the vehicle structure crushes to its maximum depth. This maximum crush depth occurs when the vehicle velocity goes to zero, the point of common velocity with the barrier. Theoretically, the maximum force occurs at the same time that the crush reaches its maximum depth.

After the approach phase is complete, the impact force drops quickly as the vehicle structure experiences a partial rebound from the maximum dynamic crush. This structural rebound has the effect of imparting a velocity to the vehicle, in the opposite direction of its initial velocity, and thus, of restoring some kinetic energy to the vehicle. When the vehicle separates from the barrier, the collision force goes to zero and the vehicle structure finishes rebounding to its final residual crush. The phase during which the vehicle rebounds from the barrier and experiences partial structural restoration is referred to as the restitution phase of the collision.

The force-dynamic crush curve from a barrier impact can be idealized as shown in Figure 1 [4, 10, 11].

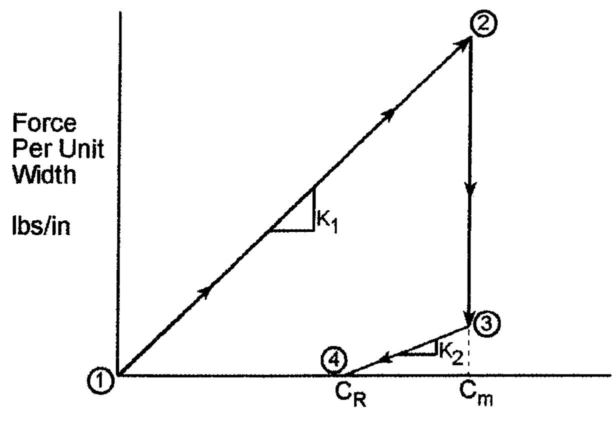

Figure 1 – Dynamic Force-Crush Curve

This force-crush curve has four points that define the structural response of the vehicle. The curve begins at Point 1, where there is zero crush and zero force. At Point 2, the curve has reached the maximum dynamic crush value C_m and a maximum force value. Between Points 2 and 3, the curve drops to a lower force level. Then, from Point 3 to Point 4, the structure experiences partial restoration to the residual crush C_R and the collision force returns to zero.[6]

There are three energy values that can be identified using the force-crush curve of Figure 1. The area underneath the line from Point 1 to Point 2 is equal to the *absorbed* energy. The area underneath the line from Point 3 to Point 4 is referred to as the *restored* energy. The difference between the absorbed and the restored energies is the *dissipated* energy (energy loss). Figure 2 shows each of these areas graphically.

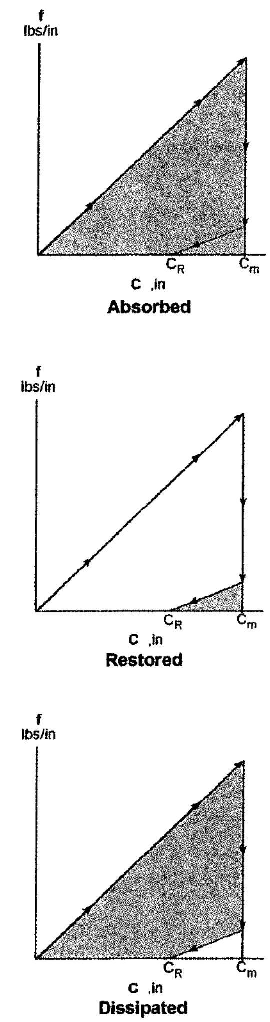

Figure 2 – Absorbed, Restored, and Dissipated Energies

Review of the derivation of Equations (1) and (2) reveals that it is the absorbed energy that should be employed in these equations. While the derivation of Equations (1) and (2) appeared in the literature in 1975, the contours of the derivation are repeated here so that its key features can be highlighted. Equations (1) and (2) are based on the one-dimensional, two degree-of-freedom mass-spring model shown in Figure 3. In this model, the masses represent the non-deforming region of each vehicle and the springs represent the deforming region of each vehicle.

The equations of motion for the masses of Figure 3 can be written in the following form:

[6] Other authors have proposed different force-crush curves (Reference 13). The discussion in this paper is applicable regardless of which idealized force-crush shape one chooses to use.

$$M_1\ddot{X}_1 = \left(\frac{K_1 K_2}{K_1 + K_2}\right)\cdot(X_2 - X_1) \quad (3)$$

$$M_2\ddot{X}_2 = \left(\frac{K_1 K_2}{K_1 + K_2}\right)\cdot(X_1 - X_2) \quad (4)$$

In Equations (3) and (4), M_1 and M_2 are the vehicle masses, X_1 and X_2 are the X-direction displacements of the masses from their original positions, $\ddot{X}_1$ and $\ddot{X}_2$ are the X-direction accelerations of the masses, and K_1 and K_2 are the stiffnesses of the springs.

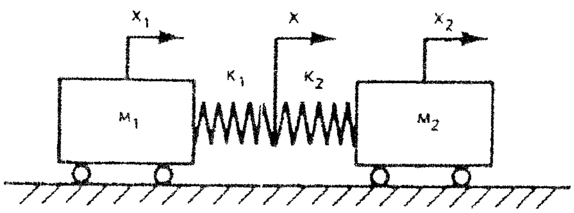

Figure 3 – The CRASH Impact Model[7]

Subtracting Equation (4) from Equation (3) and letting $\delta = X_1 - X_2$ yields Equation (5).

$$\ddot{\delta} + \Omega^2\delta = 0 \quad (5)$$

In Equation (5), ...

$$\Omega = \sqrt{\frac{M_1 + M_2}{M_1 M_2}\cdot\frac{K_1 K_2}{K_1 + K_2}} \quad (6)$$

At t = 0, the springs are uncompressed ($\delta = 0$) and the relative velocity of the masses is equal to $\dot{X}_{10} - \dot{X}_{20}$. Given these initial conditions, the solution to Equation (5) is given by Equation (7).

$$\delta = \left(\dot{X}_{10} - \dot{X}_{20}\right)\cdot\frac{1}{\Omega}\cdot\sin\Omega t \quad (7)$$

The maximum relative displacement of the vehicles is given by Equation (8) and occurs when the sine term in Equation (7) is equal to 1. This is the same time that the relative velocity between the masses goes to zero, *the time of common velocity*.

$$\delta_{\max} = \left(\dot{X}_{10} - \dot{X}_{20}\right)\cdot\frac{1}{\Omega}. \quad (8)$$

[7] Figure 3 is reproduced from Reference 9.

Now let $\delta_1 = X_1 - X$ and $\delta_2 = X - X_2$. For force equilibrium it is necessary that

$$K_1\delta_1 = K_2\delta_2. \quad (9)$$

And since $\delta = \delta_1 + \delta_2$, we can write

$$\delta_1 = \left(\frac{K_2}{K_1 + K_2}\right)\cdot\delta. \quad (10)$$

Using Equation (9), (10) and $\delta = \delta_1 + \delta_2$, Equation (8) can be written as

$$\dot{X}_{10} - \dot{X}_{20} = \sqrt{\left(\frac{M_1 + M_2}{M_1 M_2}\right)\cdot\left(K_1\delta_{1,\max}^2 + K_2\delta_{2,\max}^2\right)} \quad (11)$$

The energy *absorbed* in compressing each of the springs can be expressed as

$$E_{A,i} = \frac{1}{2}K_i\delta_{\max,i}^2. \quad (12)$$

Equation (12) is the *absorbed* energy because it occurs at the time of maximum crush, prior to any rebound of the spring that represents the vehicle structure. Therefore, Equation (11) can be written as

$$V_A = \dot{X}_{10} - \dot{X}_{20} = \sqrt{\frac{M_1 + M_2}{M_1 M_2}\cdot 2\cdot E_A} \quad (13)$$

In Equation (13), E_A is the total absorbed energy for the two-vehicle system. Equation (13) relates the total *absorbed* crush energy to the initial relative velocity, V_A, and is equivalent to Equation (1).

At the time of maximum mutual spring compression, the masses reach a common velocity, V_C. From conservation of momentum, this common velocity can be written as

$$V_C = \frac{M_1\dot{X}_{10} + M_2\dot{X}_{20}}{M_1 + M_2}. \quad (14)$$

The changes in velocity experienced by the masses, ΔV_1 and ΔV_2, from the initial time, t=0, to the time of maximum mutual spring compression are, thus, given by Equations (15) and (16).

$$\Delta V_1 = \dot{X}_{10} - V_C = \dot{X}_{10} - \left(\frac{M_1\dot{X}_{10} + M_2\dot{X}_{20}}{M_1 + M_2}\right) \quad (15)$$

$$\Delta V_2 = V_C - \dot{X}_{20} = \left(\frac{M_1 \dot{X}_{10} + M_2 \dot{X}_{20}}{M_1 + M_2} \right) - \dot{X}_{20} \qquad (16)$$

It is important to note that the velocity changes of Equations (15) and (16) are the velocity changes for the approach phase of the collision only. They do not include the restitution phase of the impact, during which there is an additional ΔV for each vehicle. Incorporating Equation (13) into Equations (15) and (16) and simplifying yields the following expression:

$$\Delta V_i = \frac{1}{M_i} \sqrt{\frac{M_1 M_2}{M_1 + M_2} \cdot 2 \cdot E_A} \qquad (17)$$

Equation (17) relates the energy *absorbed* in crushing the vehicle structure to the change in velocity experienced by each vehicle during the approach phase of the impact. It is equivalent to Equation (2).

There are two important points to observe regarding the derivation of Equations (1) and (2). First, the model of Figure 3 only considers energy absorption due to physical displacement of the vehicle structure, as modeled by the springs. Energy loss due to intervehicular sliding (friction-type energy loss) is not considered. Thus, Equations (1) and (2) are applicable to impacts where the dominant mechanism of energy loss is vehicle deformation and where intervehicular sliding is negligible. Equations (1) and (2) do not hold when energy loss due to intervehicular sliding is considered. In cases where friction-type energy losses are significant and need to be considered, Equations (1) and (2) should not be applied without modification. In these cases, a direct application of the principle of impulse and momentum is more appropriate, as discussed by Brach in Reference 1.

The second observation that should made is that the derivation of Equations (1) and (2) relies on identifying the point at which the maximum dynamic crush is achieved, the point at which the vehicle reaches a common velocity with the barrier or the other vehicle and the point of maximum energy *absorption* (Point 2 in Figure 1). Confusion may arise surrounding this point since crush analysis uses measurement of the residual (static) crush to quantify the crush energy, while the maximum energy absorption occurs at the maximum dynamic crush depth. It is natural to think of the *residual* crush being associated with the dissipated energy, as it is in Figure 2. However, it should be kept in mind that the derivation of Equations (1) and (2) is separate from the derivation of the residual crush model that actually provides the crush energy estimate. Equations (1) and (2) simply provide the physical relationships between the crush energy and the approach velocity and velocity changes. They do not provide the tool for actually quantifying that crush energy.

Equation (18) is used in crush analysis to obtain the energy value for use with Equations (1) and (2). (16)

$$E = \left(\frac{B}{2} C_R^2 + A C_R + \frac{A^2}{2B} \right) \cdot w_0 \qquad (18)$$

In Equation (18), A and B are the crush stiffness coefficients and w_0 is the damage width. The A and B stiffness coefficients can be calculated such that the residual crush measurements yield *either* the absorbed or the dissipated crush energy. This being the case, the use of residual crush measurements in the model does not automatically imply that Equation (18) will yield the dissipated energy.

It can be further demonstrated that the absorbed energy should be used in Equations (1) and (2) by writing Equation 1 in a form appropriate for a barrier impact, where the mass of the barrier approaches infinity, as follows:

$$V_A = \sqrt{\frac{2 \cdot E_A}{M_1}} \qquad (19)$$

Equation (19) can be rewritten in the following form:

$$E_A = \frac{1}{2} M_1 V_A^2 \qquad (20)$$

For the barrier impact case, Equation (20) is true by definition. However, Equation (20) would not be true if the left hand side were written as the dissipated energy, since the dissipated crush energy is instead defined as follows:

$$E_d = \frac{1}{2} M_1 V_A^2 - \frac{1}{2} M_1 V_S^2 \qquad (21)$$

In Equation (21), E_d is the dissipated energy and V_S is the maximum velocity at which the vehicle separates from the barrier.

Equations (1) and (2), therefore, properly employ the absorbed crush energy. These equations can, however, be rewritten in a form that uses the dissipated energy, since the absorbed and dissipated energies are related through the coefficient of restitution (when friction-type energy losses are negligible). This is discussed in the next section.

REWRITTING EQUATIONS (1) AND (2)

To obtain a different form of Equations (1) and (2), note the following relationships between the absorbed, restored, and dissipated energies:

$$E_d = E_A - E_R \quad (22)$$

$$\varepsilon = \sqrt{\frac{E_R}{E_A}} \quad (23)$$

Equation (22) is true by definition when friction-type energy losses are negligible and follows from the geometry of Figure 2. Equation (23) defines the coefficient of restitution in terms of the restored and absorbed energies for crush analysis. When combined with the effective mass concept [14], Equation (23) can be shown to be applicable to a general planar impact during which friction-type energy losses are negligible. Reference 15 contains a derivation of Equation (23) for such a general planar impact. When combined, Equations (22) and (23) yield the following relationship between the absorbed and dissipated energies:

$$E_A = \frac{E_d}{1-\varepsilon^2} \quad (24)$$

Substitution of Equation (24) into Equations (1) and (2) yields the following set of equations:

$$V_A = \sqrt{\frac{M_1 + M_2}{M_1 M_2} \cdot 2 \cdot \frac{E_d}{1-\varepsilon^2}} \quad (25)$$

$$\Delta V_i = \frac{1}{M_i}\sqrt{\frac{M_1 M_2}{M_1 + M_2} \cdot 2 \cdot \frac{E_d}{1-\varepsilon^2}} \quad (26)$$

Thus, while the derivation of Equations (1) and (2) dictates that the absorbed energy be input into the equations, the coefficient of restitution can be used to write a form of these equations that uses the dissipated energy. Within Equations (25) and (26), the coefficient of restitution converts the dissipated energy into the absorbed energy. *It is important to note that even though Equation 26 includes the coefficient of restitution, it does not account for the restitution phase of the impact and still only yields the approach phase velocity changes.*

For severe impacts, confusing the absorbed and dissipated energies will have little effect on the calculated closing speed and ΔVs. For severe frontal impacts, where the coefficient of restitution will usually fall between 0.1 and 0.2, the difference between the absorbed and dissipated energies will only be between 1 and 4 percent. Since this difference will fall underneath the square root signs of the equations for closing speed and ΔV, the difference in calculated values that will result from confusing the absorbed and dissipated energies will be negligible. The difference between the absorbed and restored energies would, of course, become more significant for low speed impacts. However, the significance of the difference between the absorbed and restored energies can be considered negligible from an accuracy standpoint for more severe impacts and should not be over emphasized for these cases.

The primary goal of the discussion in this paper is not, therefore, to achieve any significant improvement in the accuracy of crush analysis. Instead, this paper aims at a clear and consistent theoretical description of the use of the absorbed and dissipated energies. The value of this discussion lies primarily in gaining understanding and clarity. Such understanding and clarity becomes essential when, for instance, one attempts to understand the relationship between the equations of crush analysis and the equations of planar impact mechanics – a discussion taken up later in this paper. Beyond that, there is always value in understanding the applications and limitations of the models that one employs.

GENERALIZING CRUSH ANALYSIS

The model of Figure 3 is one-dimensional, as are the equations that it yields, Equations (1) and (2) or Equations (25) and (26). Each vehicle is allowed to translate along a single coordinate direction and rotation is not considered. The restitution phase of the impact is considered negligible and since residual crush measurements are taken perpendicular to the original vehicle side, the crush energy calculated with Equation (18) inherently assumes that the collision force acted perpendicular to the original shape of the damaged side. Equations (1) and (2) and Equations (25) and (26) are, thus, applicable to central impacts where the restored energy is negligible and the collision force acts perpendicular to the original shape of the damaged vehicle side.

How is it that the one-dimensional equations of crush analysis can be extended to the general two-dimensional (planar) impact, where each vehicle has three degrees-of-freedom – two in translation and one in rotation – where the collision forces do not act through the vehicle centers of gravity, where the collision forces are not applied perpendicular to original shape of the damaged vehicle side, and where restitution is not negligible? This general application of the crush analysis equations is accomplished, first, by incorporating the restitution phase of the impact, second, by using the *effective mass concept*, and third, by adjusting the calculated damage energy to reflect non-perpendicular collision forces.

RESTITUTION

References 10, 11 and 15 discuss the incorporation of restitution into crush analysis and that discussion will not be taken up here. Suffice it to say here that if the coefficient of restitution is incorporated into the crush